Decision Limits in Gamma-Ray Spectrometry

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Decision Limits (after Currie)

- **Critical Limit,** $L_C$
  - Is this net count significant?

- **Upper Limit,** $L_U$
  - What is the maximum statistically reasonable count in this sample?

- **Detection Limit,** $L_D$
  - What is the minimum net count I can be confident of detecting in a sample like this?

- **Minimum Detectable Activity,** $MDA$
  - What is the least activity I can be confident of detecting in a sample like this?
Decision Limits (after Currie)

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Peak Area Estimation

Define a Peak Region from $L$(lower) to $U$(upper):

\[ Gross = G = \sum_{i=L}^{U} C_i \]
Peak Area Estimation

Define a Background Regions on each side of peak - \( m \) channels wide

\[
\text{Background Area} = \sum_{i=L-m}^{L-1} C_i + \sum_{i=U+1}^{U+m} C_i
\]
Peak Area Estimation

Net Area \( A = \sum_{i=L}^{U} C_i - n \left[ \sum_{i=L-m}^{L-1} C_i + \sum_{i=U+1}^{U+m} C_i \right] / 2m \)
Peak Area Estimation

Net Area \( A = \sum_{i=L}^{U} C_i - \left[ \sum_{i=L-m}^{L-1} C_i + \sum_{i=U+1}^{U+m} C_i \right] \frac{n}{2m} \)

\[ \text{var}(A) = \text{var}(G) + \text{var}(B) \]

\[ \text{var}(A) = V = \sum_{i=L}^{U} C_i + \left[ \sum_{i=L-m}^{L-1} C_i + \sum_{i=U+1}^{U+m} C_i \right] \frac{n^2}{4m^2} \]

Note that the variance of the background is not numerically equal to the background!
Critical Limit, $L_C$

- If we measure a count near background, how can we tell if it’s real?
- Let’s start with a sample which contains NONE of the nuclide we want to measure
- If we measure the sample many, many times, all net counts will be near zero, but some will be above, some below.
- We will be able to plot the distribution of counts
Critical Limit, $L_C$

Distribution of counts when no activity present
Critical Limit, $L_C$

Distribution of counts when no activity present

We set $\alpha$ to be, say, 5% - so that at, or below, $L_C$ we would be 95% confident that we had really measured no activity.

Count NOT Significant

Count Significant
Critical Limit, $L_C$

Single Count (e.g. beta counting):

$$L_C = 1.645 \sqrt{2B} = 2.33 \sqrt{B}$$

where $B$ is the background count
Critical Limit, $L_C$

Single Count (e.g. beta counting):

\[ L_C = 1.645 \sqrt{2B} = 2.33 \sqrt{B} \]

where $B$ is the background count

Peak Area Measurement:

\[ L_C = 1.645 \sqrt{[B(1+n/2m)]} \]

where $B$ is the estimated peak background,

$n$ is the peak width and $m$ the background width
How do we use $L_C$ and $L_U$?

Sample 1: Above Critical Limit - quote result and confidence limit.

$L_C$: Critical Limit

Net Count Zero
How do we use $L_C$ and $L_U$?

Sample 1: Above Critical Limit - quote result and confidence limit

Sample 2: Below Critical Limit - Not Detected

$L_C$: Critical Limit

Net Count Zero
Upper Limit, \( L_U \)

If the count is not significant, what should we quote?
We quote a result:
\[ A \pm \sigma_A \]
For any result we can say that:
Statistically our true result will be less than
\[ A + 1.645 \sigma_A \]
on 95% of occasions
This, then, is our upper limit, \( L_U \)
Even if our count is not significant we can still quote an Upper Limit
(Why don’t we?)
How do we use $L_C$ and $L_U$

Sample 1: Above Critical Limit - quote result and confidence limit.

$L_C$: Critical Limit

Net Count Zero
How do we use $L_C$ and $L_U$?

Sample 1: Above Critical Limit - quote result and confidence limit

Sample 2: Below Critical Limit - quote < Upper Limit

$L_C$: Critical Limit

Net Count Zero
Detection Limit, $L_D$

What is the minimum number of counts I can be confident of detecting?

(That is NOT the same as asking ‘What is the maximum number of counts which I could have in this sample?’)

Let’s assume we have a sample containing precisely the amount of activity to give that number of counts

Measure the sample many, many times and plot the frequency distribution of the net count
Detection Limit, $L_D$

- Will NOT be detected
- 95% Will be detected

$\beta$: Probability density function

$k_\beta \cdot \sigma_D$: Detection limit

$\sigma_D$: Standard deviation

$k_\beta$: Normalized factor
Detection Limit, $L_D$

$\sigma_0$

$\alpha$

$\beta$

$k_\beta \cdot \sigma_D$

Will NOT be detected

95% Will be detected
Detection Limit, $L_D$

Single Count (e.g. beta counting):

$$L_D = 2.71 + 3.29 \sqrt{2B}$$

where $B$ is the background count

Peak Area Measurement:

$$L_D = 2.71 + 3.29 \sqrt{B(1+n/2m)}$$

where $B$ is the estimated peak background,
$n$ is the peak width and $m$ the background width
Minimum Detectable Activity, *MDA*

- *MDA* is normally calculated as the activity equivalent of the Limit of Detection.
- It answers the question ‘What is the minimum activity I can be 95% confident of detecting?’
- That is not the same as ‘What is the maximum activity which could be in this sample?’
- *MDAs* are often calculated incorrectly by using the single count equation rather than the peak area equation.
- If the MDA calculation doesn’t take into account how the peak background is estimated it must be wrong.
Minimum Detectable Activity, \textit{MDA}

- It is possible to measure activity below the \textit{MDA} i.e. The \textit{MDA} is NOT the \textit{MAD}!

- If there were the \textit{MDA} amount of activity present there would be a peak in the spectrum (in 95% of instances)

- If the activity (or peak) is not significant (i.e. below the Critical Limit) then there must be less than the \textit{MDA} present

- Upper Limit will always be less than \textit{MDA} - an effortless improvement in sensitivity!

- Why calculate \textit{MDA} even when a peak is present?

- Why not use the correct equation?
Channel

Counts per channel

Critical Limit
Detection Limit
L_c < Ct > L_d

1,000 cts/ch
3,000 cts/ch
10,000 cts/ch
30,000 cts/ch
Counts per channel

Channel

Critical Limit
Detection Limit
L_c < C_t > L_d

1,000 cts/ch 3,000 cts/ch
Summary:

- *MDA*, as normally calculated, is not the Minimum Activity Detectable
- It represents a theoretical ‘What might be achieved?’
- *Should be used* when assessing a method and when quoting performance (tenders?).
- To avoid all confusion the term should be scrapped and a new one devised.
- *When peaks not present*, calculate *Upper Limit*
- *...which represents what has been achieved.*